Paper Reference(s)

6680/01

Edexcel GCE

Mechanics M4

Advanced

Tuesday 22 June 2010 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M4), the paper reference (6680), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 6 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. [*In this question* **i** *and* **j** *are unit vectors due east and due north respectively*.]

A man cycles at a constant speed u m s^{-1} on level ground and finds that when his velocity is $u\mathbf{j}$ m s^{-1} the velocity of the wind appears to be $v(3\mathbf{i} - 4\mathbf{j})$ m s^{-1}, where v is a positive constant.

When the man cycles with velocity $\frac{1}{5}u(-3\mathbf{i} + 4\mathbf{j})$ m s^{-1}, the velocity of the wind appears to be $w\mathbf{i}$ m s^{-1}, where w is a positive constant.

Find, in terms of u, the true velocity of the wind.

(7)

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2. Two smooth uniform spheres S and T have equal radii. The mass of S is 0.3 kg and the mass of T is 0.6 kg. The spheres are moving on a smooth horizontal plane and collide obliquely. Immediately before the collision the velocity of S is $\mathbf{u}_1$ m s^{-1} and the velocity of T is $\mathbf{u}_2$ m s^{-1}. The coefficient of restitution between the spheres is 0.5. Immediately after the collision the velocity of S is $(-\mathbf{i} + 2\mathbf{j})$ m s^{-1} and the velocity of T is $(\mathbf{i} + \mathbf{j})$ m s^{-1}.

Given that when the spheres collide the line joining their centres is parallel to **i** ,

(*a*) find

(i) $\mathbf{u}_1$,

(ii) $\mathbf{u}_2$.

(6)

After the collision, T goes on to collide with a smooth vertical wall which is parallel to **j**.

Given that the coefficient of restitution between T and the wall is also 0.5, find

(*b*) the angle through which the direction of motion of T is deflected as a result of the collision with the wall,

(5)

(c) the loss in kinetic energy of T caused by the collision with the wall.

(3)

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3. At 12 noon, ship A is 8 km due west of ship B. Ship A is moving due north at a constant speed of 10 km h^{-1}. Ship B is moving at a constant speed of 6 km h^{-1} on a bearing so that it passes as close to A as possible.

(*a*) Find the bearing on which ship B moves.

(4)

(*b*) Find the shortest distance between the two ships.

(3)

(*c*) Find the time when the two ships are closest.

(3)

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4. A particle of mass m is projected vertically upwards, at time $t = 0$, with speed U. The particle is subject to air resistance of magnitude $\frac{mgv^2}{k^2}$, where v is the speed of the particle at time t and k is a positive constant.

(*a*) Show that the particle reaches its greatest height above the point of projection at time $\frac{k}{g} \tan^{-1}\left(\frac{U}{k}\right)$

(6)

(*b*) Find the greatest height above the point of projection attained by the particle.

(6)

5.

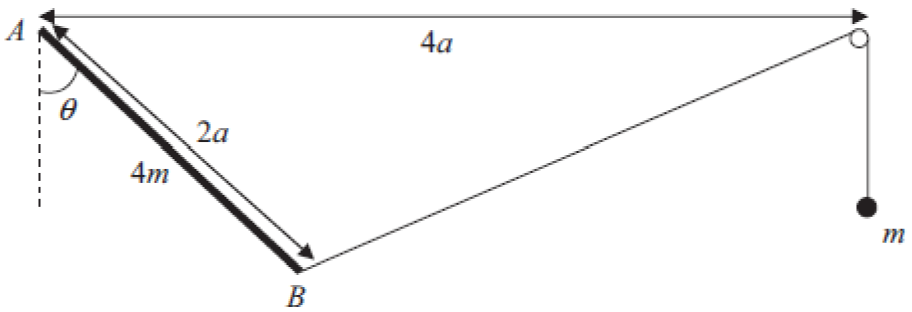

Figure 1

The end A of a uniform rod AB, of length $2a$ and mass $4m$, is smoothly hinged to a fixed point. The end B is attached to one end of a light inextensible string which passes over a small smooth pulley, fixed at the same level as A. The distance from A to the pulley is $4a$. The other end of the string carries a particle of mass m which hangs freely, vertically below the pulley, with the string taut. The angle between the rod and the downward vertical is θ, where $0 < \theta < \frac{\pi}{2}$, as shown in Figure 1.

(*a*) Show that the potential energy of the system is

$$2mga(\sqrt{(5 - 4\sin\theta)} - 2\cos\theta) + \text{constant.}$$

(5)

(*b*) Hence, or otherwise, show that any value of θ which corresponds to a position of equilibrium of the system satisfies the equation

$$4\sin^3\theta - 6\sin^2\theta + 1 = 0.$$

(5)

(*c*) Given that $\theta = \frac{\pi}{6}$ corresponds to a position of equilibrium, determine its stability.

(5)

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6. Two points A and B lie on a smooth horizontal table with $AB = 4a$. One end of a light elastic spring, of natural length a and modulus of elasticity $2mg$, is attached to A. The other end of the spring is attached to a particle P of mass m. Another light elastic spring, of natural length a and modulus of elasticity mg, has one end attached to B and the other end attached to P. The particle P is on the table at rest and in equilibrium.

(*a*) Show that $AP = \dfrac{5a}{3}$.

(4)

The particle P is now moved along the table from its equilibrium position through a distance $0.5a$ towards B and released from rest at time $t = 0$. At time t, P is moving with speed v and has displacement x from its equilibrium position. There is a resistance to motion of magnitude $4m\omega vg$ where $\omega = \sqrt{\left(\dfrac{g}{a}\right)}$.

(*b*) Show that $\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} + 4\omega\dfrac{\mathrm{d}x}{\mathrm{d}t} + 3\omega^2 x = 0$.

(5)

(*c*) Find the velocity, $\dfrac{\mathrm{d}x}{\mathrm{d}t}$, of P in terms of a, ω and t.

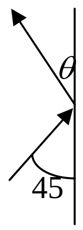
(8)

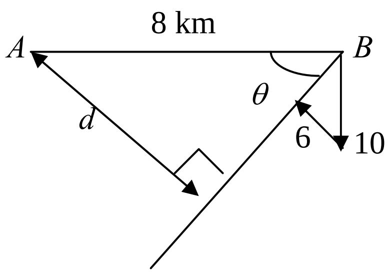
TOTAL FOR PAPER: 75 MARKS

END

Summer 2010
Mechanics M4 6680
Mark Scheme

Question Number	Scheme	Marks
Q1	$v(3\mathbf{i}-4\mathbf{j})=\mathbf{v}_W-u\mathbf{j}$	M1A1
	$\mathbf{v}_W=3v\mathbf{i}+(u-4v)\mathbf{j}$	
	$w\mathbf{i}=\mathbf{v}_W-\frac{u}{5}(-3\mathbf{i}+4\mathbf{j})$	M1A1
	$\mathbf{v}_W=(w-\frac{3u}{5})\mathbf{i}+\frac{4u}{5}\mathbf{j}$	
	$(u-4v)=\frac{4u}{5}$	M1
	$v=\frac{u}{20}$	A1
	$\mathbf{v}_W=\frac{3u}{20}\mathbf{i}+\frac{4u}{5}\mathbf{j}$	A1
		7

Question Number	Scheme	Marks
Q2		
(a)	$\uparrow 2$ $\quad\quad \uparrow 1$ $1 \leftarrow \quad\quad \rightarrow 1$ S 0.3kg $\quad$ T 0.6 kg $2 \uparrow \quad\quad \uparrow 1$ $\rightarrow v \quad\quad w \leftarrow$	
	$0.3v - 0.6w = 0.3$	M1 A1
	$v - 2w = 1$	
	$\frac{1}{2}\ (v + w) = 2$	M1 A1
	$v + w = 4$	
	$w = 1, v = 3$	
	(i) $\mathbf{u_1} = 3\mathbf{i} + 2\mathbf{j}$ (ii) $\mathbf{u}_2 = -\mathbf{i} + \mathbf{j}$	A1 A1
		(6)
(b)	$\uparrow 1$ $v \leftarrow$	
	$v\ = 0.5$	B1
	$1 \uparrow$ $\rightarrow 1$	
		
	$\tan\theta =\ 0.5$ $\quad\quad$ $\tan\theta$ = their v	M1
	$\theta = 26.6$	A1
	their θ + 45°	M1
	Defln angle $= 45 + 26.6 = 71.6^{\circ}$	A1
		(5)
(c)	KE Loss $= \frac{1}{2}$ x 0.6 x $\left\{(1^2 + 1^2) - (1^2 + v^2)\right\}$	M1 A1
	$= 0.225$ J	A1
		(3)
		14

Question Number	Scheme	Marks
Q3 (a)		M1
	$\cos\theta = \frac{6}{10} \Rightarrow \theta = 53.1^{\circ}$	M1 A1
	Bearing is 307°	A1
		(4)
(b)	$d = 8\sin\theta$ $(=8\times0.8)$ $= 6.4$ km	M1 A1 A1
		(3)
(c)	$T = \frac{8\cos\theta}{\sqrt{10^2 - 6^2}}$	M1 A1
	$= 0.6$ hrs i.e. the time is 12:36 pm	A1
		(3) **10**

Question Number	Scheme	Marks
Q4		
(a)	$-mg(1+\frac{v^2}{k^2}) = m\frac{\mathrm{d}v}{\mathrm{d}t}$	M1 A1
	$g\int_0^T \mathrm{d}t = \int_U^0 \frac{-k^2\mathrm{d}v}{(k^2+v^2)}$	DM1
	$T = \frac{k}{g}\left[\tan^{-1}\frac{v}{k})\right]_0^U$	A1
	$= \frac{k}{g}\tan^{-1}\frac{U}{k}$	DM1A1
		(6)
(b)	$-mg(1+\frac{v^2}{k^2}) = mv\frac{\mathrm{d}v}{\mathrm{d}x}$	M1 A1
	$g\int_0^H \mathrm{d}x = \int_U^0 \frac{-k^2v\mathrm{d}v}{(k^2+v^2)}$	DM1
	$H = \frac{k^2}{2g}\left[\ln(k^2+v^2\right]_0^U$	A1
	$H = \frac{k^2}{2g}\ln\frac{(k^2+U^2)}{k^2}$	DM1A1
		(6)
		12

Question Number	Scheme	Marks
Q5		
(a)	$\sqrt{4a^2+16a^2-16a^2\sin\theta}$	M1 A1
	Let length of string be L.	
	$V=-4mga\cos\theta-mg(L-\sqrt{4a^2+16a^2-16a^2\sin\theta}$	M1 A1
	$=-4mga\cos\theta-mgL+2mga\sqrt{5-4\sin\theta}$	
	$=2mga\left\{\sqrt{5-4\sin\theta}-2\cos\theta\right\}+\text{constant}$ **	A1
		(5)
(b)	$V'(\theta)=2mga\left\{\dfrac{-2\cos\theta}{\sqrt{5-4\sin\theta}}+2\sin\theta\right\}$	M1 A1
	For equilibrium, $V'(\theta)=0$	
	$\left\{\dfrac{-2\cos\theta}{\sqrt{5-4\sin\theta}}+2\sin\theta\right\}=0$	M1
	$\dfrac{\cos^2\theta}{5-4\sin\theta}=\sin^2\theta$	
	$1-\sin^2\theta=\sin^2\theta(5-4\sin\theta)$	DM1
	$4\sin^3\theta-6\sin^2\theta+1=0$ **	A1
		(5)
(c)	$V''(\theta)=2mga(\dfrac{\left\{\sqrt{5-4\sin\theta}.2\sin\theta-\dfrac{-2\cos\theta.(-4\cos\theta)}{2\sqrt{5-4\sin\theta}}\right\}}{(5-4\sin\theta)}+2\cos\theta)$	M1 A1 A1
	$V''(\dfrac{\pi}{6})=2mga\left\{\dfrac{\sqrt{3}-\dfrac{8\text{x}\frac{3}{4}}{2\sqrt{3}}}{3}+\sqrt{3}\right\}=2mga\sqrt{3}>0$ so stable	DM1 A1
		(5)
		15

Question Number	Scheme	Marks
Q6		
(a)	$T_1 = \frac{2mge}{a}$; $T_2 = \frac{mg(2a-e)}{a}$	B1 (either)
	$T_1 = T_2$	
	$2e = (2a-e)$	M1 A1
	$e = \frac{2a}{3}$	
	$AP = a + \frac{2a}{3} = \frac{5a}{3}$ **	A1
		(4)
(b)	$T_2 - T_1 - 4m\omega\dot{x} = m\ddot{x}$	
	$\frac{mg}{a}\left(\frac{4a}{3} - x\right) - \frac{2mg}{a}\left(\frac{2a}{3} + x\right) - 4m\omega\dot{x} = m\ddot{x}$	M1 A3
	$\ddot{x} + 4\omega\dot{x} + \frac{3g}{a}x = 0$	
	$\ddot{x} + 4\omega\dot{x} + 3\omega^2 x = 0$ **	A1
		(5)
(c)	$\lambda^2 + 4\omega\lambda + 3\omega^2 = 0$	
	$(\lambda + 3\omega)(\lambda + \omega) = 0$	M1
	$\lambda = -3\omega$ or $\lambda = -\omega$	
	$x = Ae^{-\omega t} + Be^{-3\omega t}$	A1
	$\dot{x} = -\omega Ae^{-\omega t} - 3\omega Be^{-3\omega t}$	M1 A1
	$t = 0$, $x = \frac{1}{2}a$, $\dot{x} = 0$	M1
	$\frac{1}{2}a = A + B$	
	$0 = -\omega A - 3\omega B$	A1
	$A = \frac{3}{4}a$, $B = -\frac{1}{4}a$	A1
	$\dot{x} = v = \frac{3}{4}a\omega\,(\mathrm{e}^{-3\omega t} - \mathrm{e}^{-\omega t})$	A1
		(8)
		17